Distributed SC-FDMA Resource Allocation Algorithm based on the Hungarian Method

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Abstract—In this paper we focus on a SC-FDMA system’s resource allocation problem considering the uplink of the 3GPP Long Term Evolution system. We first define a utility function at each sector aiming at maximizing sum of average SINR, and implement the fairness factor which enables the resource allocator to schedule resources fairly among the users. Then, for the fairness-aware utility function, the optimization problem is solved using the iterative Hungarian. In multi-cell system, as the users suffer a uncorrelated inter-cell interference, it is difficult to find the globally optimal radio resource allocation. We propose a distributed allocation method which avoids the drastic interference level changes. Results exhibit that the proposed allocation method has a robustness to the interference variation and can be close to the globally optimal allocation.

Index Terms—OFDMA, SC-FDMA, Resource Allocation, Multi-Cell, Inter-Cell Interference

I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) is considered as a promising radio access technology for the 4th generation (4G) mobile communication system. By transmitting data stream through a numerous orthogonal subcarriers, it enables high-rate transmission even in a frequency-selective fading channel. Also it is robust against the intersymbol interference (ISI) even in a very dispersive channel due to its long symbol duration. For these reasons, OFDMA has been considered as the downlink (from base station to mobile station) radio access scheme in several air-interface standards including IEEE802.16e[1] and 3GPP LTE[2] system. However, due to the high Pick-to-Average-Power-Ratio (PAPR) of the OFDM signal, implementing the OFDM in the uplink (from mobile station to base station) is less attractive – the mobile station has a strictly limited transmit power compared to the base station. Being motivated by this high PAPR problem, the single-carrier frequency division multiple access (SC-FDMA), more specifically so-called DFT(Discrete Fourier Transform)-spread OFDM, has been chosen as the uplink access scheme in 3GPP LTE system. In the DFT-spread OFDM, a DFT block is added prior to the conventional OFDM operation blocks, as depicted in Fig. 1, where the data symbols are spread over all the subcarriers and each subcarrier contains the information of all transmitted symbols. By doing this, it produces virtually a single carrier signal with a low PAPR, but it maintains the characteristic of OFDMA.

In the OFDMA-based cellular system, the uplink resource allocation problem boils down to assigning the orthogonal time-frequency resources to the multiple users. While the OFDMA-based system is able to preserve the orthogonality in the multi-path fading channel and thereby eliminate the intra-cell interference, the inter-cell interference (ICI) caused by the users in the neighboring cell transmitting at the same frequency band, becomes a critical issue. Therefore, when designing the resource allocation scheme for the uplink of OFDMA cellular systems, the consideration of the interferences from the users in the neighboring cells becomes very important. However, it is non-trivial to directly control the ICI and this difficulty is aggravated with the unpredictability of the time-varying ICI. Although the resource allocation algorithm assigns some of radio channels to a user based on the measured channel quality or the ICI of the present, due to the inherent delay between the measurement and the assignment, the actual channel quality can be different. Furthermore, assuming the distributed implementation, the changes of resource allocation in one cell can perturb other neighboring cells’ resource allocation, then reaching the global optimality in the resource allocation becomes a challenging issue.

Resource allocation problem for the multi-cell OFDMA systems has been studied in various works [3][4][5][6][7]. The proportional fair scheduling with logarithmic user data rate is introduced in [3]. [4] proposes a search-tree based algorithm and it gives a significant gain over a random allocation. However, it does not achieve the optimal allocation. In [5], three allocation algorithms (greedy, maximum greedy and Hungarian algorithm) are investigated and it is claimed that the Hungarian algorithm outperforms the other two algorithms in a single-cell scenario. Some studies consider the resource allocation multi-cell system [6], [7]. In [6], the resource allocator using the linear programming aiming at minimizing transmission power converges to a stable interference state as...
well as the stable resource allocation. However, it is achieved by removing assigned sub-carriers from cell-edge user until it reaches to a stable state, which results in sacrificing the cell-edge user throughput. In [7], authors propose a SC-FDMA allocation scheme to allocate more resources to the users in the base station’s vicinity than the ones at the cell boundary. It can lead to a reduction of interference by sacrificing cell-edge users’ spectral efficiency as they are likely to cause stronger interference.

We introduce a SC-FDMA resource allocation algorithm to find an optimal solution for maximizing total adjusted Signal to Interference plus Noise Ratio (SINR) while adapting fairness factor in order to consider the fairness as well. Additionally, we propose a cell-alternating scheduling algorithm across the multiple cells. The alternating implementation of the scheduling algorithm tries to reduce the unpredictability and the time-varying interference, and by doing so it can minimize the edge-to-edge conflict among the users in the neighboring cells. We show how the proposed algorithm can increase the system throughput and the cell-edge user throughput through the simulation.

Rest of the paper is organized as follow. Section II shows the overall system model, and the detailed per-sector allocation algorithm is described in Section III. Section IV explains the cell-alternating resource allocation algorithm in the multi-cell system. Section V illustrates the evaluation methodology used for the performance evaluation and Section VI presents the results of the simulation. Finally, Section VII draws the conclusion and discusses future work.

II. SYSTEM MODEL

Consider a multi-cell LTE uplink cellular system as shown in Fig. 2. It is assumed that the multiple users are attached to one of the sectors and each Base Station (BS) in a sector does not communicate each other regarding the information of the other cell’s resource allocation and the channel qualities of users in other cells. We assume that all the sectors are allowed to use any part of the bandwidth, in other words the frequency reuse factor is one, to maintain the large spectral efficiency. Each user in a sector transmits signal to serving BS while simultaneously generating interference to other sectors’ BS.

An OFDM physical resource block (PRB) consists of 12 consecutive OFDM sub-carriers during one transmission time slot. Total bandwidth is divided into $N_{PRB}$ of PRBs and we ignore bandwidth needed for control channel. Then, we consider the consecutive bandwidth characteristic of SC-FDMA. SC-FDMA needs the PRBs to be allocated for each user in a contiguous manner in order to maintain the single-carrier property. We define the Resource Set (RS) which consists of a set of consecutive PRBs. We assume that the number of PRBs constituting one RS is the same for all RSs in a sector and it is computed by dividing total available PRBs by the number of users to transmit. When the number of PRBs is not divided by the number of users, the remaining PRB’s belong to the last RS. The maximum number of RSs is set as 10. If the number of users exceeds 10, the resource allocator randomly selects 10 users to transmit. Fig. 3 shows how the PRB and RS are formed.

Assuming that the multi-path fading’s magnitudes and phases are constant over one PRB, the uplink SINR for user $k$ on PRB $n$ is given by

$$\text{SINR}_{(k,n)} = H_k(n) \times \bar{G} \times \left( \frac{N}{N + N_p} \right) \times \frac{R_D}{N_{SD}/N_{ST}}, \quad (1)$$

which is the function of channel’s current frequency response on PRB $n$ ($H_k(n)$), the current geometry ($\bar{G}$), the FFT size ($N$), the cyclic prefix length ($N_p$), the percentage of the maximum total available transmission power allocated to the data sub-carriers ($R_D$), the number of data sub-carriers per Transmission Time Interval (TTI) ($N_{SD}$), and the number of total useful sub-carriers per TTI ($N_{ST}$). Since the uplink system is considered, the geometry on PRB $n$ for user $k$ can be computed as

$$\bar{G} = \frac{P_{TX}(k) \times P_{L}(k,j)}{BN_0 + I_j(n)}, \quad (2)$$

where $BN_0$ is thermal noise power on one PRB, $P_{TX}(k)$ is the transmission power of user $k$ for one PRB, $j$ is the index of user $k$’s serving BS, and $P_{L}(k,j)$ is the total pathloss (or the static gain) between user $k$ and BS $j$. $I_j(n)$ in (2) represents the interference, and it is expressed as

$$I_j(n) = \sum_{l \in U(n), l \neq k} P_{TX}(l) \times P_{L}(l,j), \quad (3)$$

where $U(n)$ is the set of users associated to BS $n$. The total power of all users in $U(n)$ is $P_{TX}(n)$.
where $U(n)$ is the set of users transmitting on PRB $n$ in the other cells, $P_{TX}(l)$ is its transmission power, and $PL_{(i,j)}$ is its total pathloss to BS $j$. In the uplink systems, it is common that the amount of interferences $(I_{j}(n))$ from the users located at the neighboring cells exceeds the amount of the thermal noise ($BN_0$) at the receiving BS. Furthermore, it is difficult to estimate the amount of the interference at the BS.

In a multi-cell distributed system, the changes in the allocation at one cell affect the neighboring cells’ allocations. This implies that the SINR assumption can be changed by the variation of the interference from neighboring cells. For example, suppose that the resource allocation algorithm in cell $A$ assigns a certain bandwidth to user $a$, since it has the best SINR and the modulation and coding scheme is selected based on the assumed SINR. On the other hand, the algorithm in cell $B$, adjacent to cell $A$, can assign the same bandwidth to cell-edge user $b$ who emits a strong interference. Then, user $a$ has to experience a poor SINR and the selected modulation and coding scheme is no longer suitable.

### III. PER-SECTOR RESOURCE ALLOCATION ALGORITHM

In this section, the per-sector resource allocation algorithm is described. First, the utility function to maximize the sum of the adjusted SINR’s is formulated. The average SINR of RS $m$ for user $k$ is given by

$$\text{SINR}^{(k,m)}_n = \frac{1}{M} \sum_{n=1, \ldots, M+1} \text{SINR}_{(k,n)}^{(n)}$$

where $i$ is the index of the first PRB on the RS, and $M$ is the number of PRBs in the RS. In the utility function, a fairness value is multiplied to every average SINR’s to balance the system throughput and the cell-edge user throughput. This metric is based on the power control algorithm in [12] and it is originally introduced to compensate the path loss. Fairness value is defined by the product between the fairness factor that we assume a constant in a system and the user’s total path loss to the serving BS. The adjusted SINR that is going to be maximized, is defined as

$$\text{SINR}''_{(k,m)} = \text{SINR}^{(k,m)} \times \Delta_{\text{fairness}}$$

where $\theta$ is the fairness factor which decides how fairly the resource allocation is performed and $PL_{(k,j)}$ is the total path loss to BS $j$ from user $k$ whose serving BS is $j$. It is a simple method for compensating path loss for the cell-edge user, and it can also be used to give an advantage to the cell-edge users in the allocation algorithm. If a user is located in a cell edge and has a large path loss value, the user’s fairness value becomes large. Then, the allocation is executed with setting a high priority to this cell-edge user. A large value of $\theta$ ensures more fairness by sacrificing the total system throughput.

The optimization problem is defined by the adjusted SINR as follow.

$$\text{maximize} \quad \sum_{k=1}^{N_{RS}} \sum_{m=1}^{N_{RS}} \text{SINR}''_{(k,m)} x_{k,m}$$

subject to

$$\sum_{k=1}^{N_{RS}} x_{k,m} = 1 \quad \text{for} \quad m = 1, 2, \ldots, N_{RS}$$

$$\sum_{m=1}^{N_{RS}} x_{k,m} = 1 \quad \text{for} \quad k = 1, 2, \ldots, N_{RS}$$

$$x_{k,m} \in \{0,1\}$$

$x_{k,m}$ takes the value 1 when user $k$ is assigned to RS $m$ otherwise 0. Each user must be assigned to one and only one RS, and each RS must have one assigned user. Then, the optimization problem can be seen as one to one assignment problem. Conventional scheduling algorithms for OFDMA system are difficult to be applied in this allocation problem due to the constraint that only one user has to be assigned to only one RS.

In [4] and [5], optimization problem in (6) is solved by the algorithm based on arranging the users and the RS’s in a matrix with the corresponding metric values and the matrix is used as the input of the algorithm. We solve the optimization problem using the Hungarian algorithm [5][8]. It provides an efficient and low complexity method to solve the one-to-one assignment problem.

![Fig. 4. Cost Metric for the Hungarian Method. Note that $N_{RS} = K$](image-url)

Two-dimensional square cost metric is used as an input to the algorithm, and it can be constructed as in Fig. 4 with the size of $N_{RS} \times N_{RS}$. 6-step algorithm in [10] is applied per-sector basis and it is a modified form from the original Munkres’ Assignment Algorithm (also known as the Hungarian Algorithm). The algorithm converts the cost metric into a series of equivalent cost metrics by manipulating rows and columns through the additions and the subtractions. It continues converting the cost metrics until it reaches a state where an optimal assignment is obvious. When the algorithm finishes, the final equivalent cost metric is consisting of non-zero or zero elements. Zero elements at the matrix after the iterations, imply that the assignment should be done for the corresponding zero-element pair. Due to the space limitation, a specific implementation is not presented in this paper.
IV. ALTERNATING RESOURCE ALLOCATION FOR MULTIPLE CELLS

This section addresses the problem of the resource allocation in a multi-cell system. The multi-cell resource allocation problem can be considered as a centralized problem, however, not only the global optimality is not known but also a large amount of overhead information makes the approach impractical. In a distributed manner, if we can expect scheduling in neighboring cells, it can be beneficial to perform a distributed optimal allocation. However, as it is impossible to correctly predict neighboring cell’s allocation or interference level, we propose a cell-alternating\(^1\) allocation which is to perform allocation by rotation among neighboring cells.

For example, consider the cell layout as in Fig. 5. Three sectors in one cell compose a rotating allocation group. First sectors in the rotation are numbered as 1 and the allocations for those sectors are performed, while the other neighboring sectors numbered as 2 and 3 maintain the previous allocation. After one transmission time interval (TTI), sectors numbered as 2 do the allocation in same way, and so do sectors numbered as 3 in next TTI. By doing so, we can avoid a sudden change of the neighboring sectors’ interferences. On the other hand, since the same numbered sectors change allocation at the same time, they cannot predict the interference level from each other. However, their distances are far enough to ignore their influence. It can be considered to group with more sectors than three in order to have farther distance among simultaneous allocation sectors. However, if the rotation group becomes bigger, the time for sustaining allocation become longer. This results in being vulnerable to fast fading channels.

\(^1\)The proposed algorithm, actually, alternates the sectors.

V. SYSTEM-LEVEL SIMULATION

The performance of the proposed algorithm is evaluated by the system level simulation according to the guidelines in [2].

The system level simulation is done with the channel model described in [11]. 19 cells and 3 sectors in one cell are assumed in the cell layout with wrap-around property. At the beginning of a simulator run, a certain number of users are uniformly distributed in 57 sectors. Then, we compute the static gain including antenna gain, path loss and shadowing between each user and sectors. Each users selects its sector with the largest static gain.

An adaptive modulation and coding is used in the simulation and the type of modulation and coding scheme is selected based on the SINR estimations over the allocated bandwidth. We assume a perfect channel quality estimation through the sounding reference signal for every TTI. The Exponential Effective SINR Mapping (EESM) [2] is used to compute the effective SINR and the Block Error Rate (BLER) curve computed from the link-level-simulation is referred to decide transmission success and failure for the effective SINR. The scheduled user sets its total transmission power using the following [12]

\[
P = \min \{ P_{\text{max}}, 10 \cdot \log_{10} M + P_0 + \alpha \cdot PL + \Delta_{\text{mcs}} \} \tag{7}
\]

where \(P_{\text{max}}\) is the maximum transmit power from the user, \(P_0\) is the power to be contained in one PRB, \(\alpha\) is the path loss compensation factor, \(PL\) is the path loss, \(M\) is the number of PRBs assigned to the user, and \(\Delta_{\text{mcs}}\) is modulation and coding dependent value signaled from the base station. The closed loop power control is not considered. Details of the simulation parameter are described in Table I.

![Fig. 5. 19 Cell's Sector Layout](image)

### TABLE I

<table>
<thead>
<tr>
<th>SIMULATION PARAMETER</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Time</td>
<td>500ms/drop, 10 drops</td>
</tr>
<tr>
<td>Layout Scenario</td>
<td>Inter-Sector Distance</td>
</tr>
<tr>
<td>Traffic Model</td>
<td>19 cells - 3 sectors/cell (Wrap-around)</td>
</tr>
<tr>
<td>Thermal Noise Density</td>
<td>Urban Macro(500m)</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>Urban Micro(200m)</td>
</tr>
<tr>
<td>Available MCS</td>
<td>Rural Macro(1732m)</td>
</tr>
<tr>
<td>HARQ</td>
<td>Full buffer</td>
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<tr>
<td>BLER Target</td>
<td>-174dBm</td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Max user power</td>
<td>10MHz - 50 PRBs</td>
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<tr>
<td>Standard deviation of slow fading</td>
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</tr>
<tr>
<td>(\alpha, P_0) (Power Control)</td>
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<tr>
<td>Propagation model</td>
<td>0.6 dB</td>
</tr>
<tr>
<td>Fast fading model</td>
<td>-57 dBm</td>
</tr>
</tbody>
</table>

VI. SIMULATION RESULT

The performance of the proposed algorithm is evaluated using the average sector throughput and the 5% user’s throughput. Our interest is how fairly the radio resource allocation is performed according to the fairness factor, and there are many ways of evaluating the fairness. We take minimum 5% user’s throughput from [11]. We simulate both the simultaneous and alternating allocation methods for all 4 scenarios from [11]. We also vary the fairness factor \(\theta\) from 0 to 1.5. Fig. 6 and 7 show how the average sector throughput and the 5% user’s throughput vary according to \(\theta\). In the case of the
while the response of cell-alternating allocation to \( \theta \) simultaneous allocation, the throughput hardly varies for \( \theta \) while the response of cell-alternating allocation to \( \theta \) is quite clear. It is because the cell-alternating allocation is better to track the neighboring sector’s interference and to have more stable interference. At the same time, it shows how the interference variation has a large impact on the radio resource allocation. This is more evident in Table II where we can see 28.8% increase of the average sector throughput with setting \( \theta \) to 0 compared to the simultaneous case. On the other hand, we can see 45.8% increase of the 5% user throughput with setting \( \theta \) to 1.5. It can be stated that the cell-alternating algorithm is better to follow up to aim for the allocation in the fairness aspect.

VII. CONCLUSION AND FUTURE WORK

In this paper, we introduced a SC-FDMA multi-cell resource allocation algorithm while adapting the fairness in the optimization problem. In an OFDMA system, ICI is the major problem and its unpredictability causes the system not to reach an optimal solution. Our proposed cell-alternating allocation algorithm is shown to be better to cope with the unpredictable interference variation. When we consider a response to the fairness factor change, the simulation results show the proposed cell-alternating allocation converges to a local optimality and it provides a better performance compared to the simultaneous allocation. However, it was not proved which the global optimal solution is and how close the proposed local optimal solution is to the global optimality. Further studies will be focused on the convergence and the proximity to the globally optimal solution.

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